OPTIMIZATION OF A SYSTEM OF LAMELLAR-VACUUM INSULATION + COOLED SHIELDS

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It is shown that the heat influx through a stack of lamellar-vacuum insulation, alternating with cooled shields, can be reduced substantially by an optimum choice of the insulation packing density and the thickness of the layers between shields.

Lamellar-vacuum insulation (LVI), which is used extensively in cryogenic engineering, is a multilayer stack, consisting of heat shields separated by inserts. Heat transfer between neighboring layers (heat shields) occurs simultaneously by radiation and thermal conduction through the material of the insert and residual-gas molecules. The efficiency of LVI depends to a considerable degree on the residual-gas pressure in the space between layers. The shields are perforated to reduce this pressure.

According to the experimental data of [1, 2] the thermal conductivity of LVI is virtually independant of the thickness of the stack but is determined by the layer-packing density and the boundary temperatures of the stack. When the number of layers increases the radiative heat transfer per unit thickness of insulation decreases and the thermal conduction grows. The effective thermal conductivity, therefore, should reach a minimum at some optimum packing density. This is also confirmed by the results of theoretical studies [3,4].

When LVI is used in conjunction with shields cooled by exhaust cryogen vapor from the cryostating zone the problem arises of determining both the optimum arrangement of shields and optimum packing of layers of insulation between them. The problem of the optimum arrangement of cooled shields was considered in [5]. In this paper, besides that we also solve the problem of optimum LVI array.

We consider parallel cold and hot surfaces at the temperatures T_c and T_h, respectively. The cold boundary is a boundary of the cryostating zone. Between the two surfaces are N shields (see Fig. 1), which are cooled successively by exhaust cryogen vapor from the cryostating zone. The spaces in this system are filled witeh LVI. In particular, if the LVI packing density $\rho_i = 0$, it is assumed that there is no LVI in the interval. The total size of the intervals is L.

The temperature of the cold boundary surface is assumed to be equal to the saturation temperature of the cryogen in the cryostating zone while the temperatures of the cooled

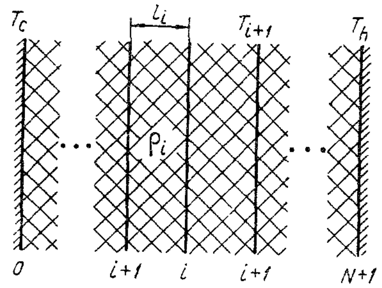

Fig. 1. Planar heat-insulating system of lamellar-vacuum insulation and cooled shields.

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shields are assumed to be uniform. The head exchage of the shields with the cryogen flows is ideal, i.e., the cryogen leaving a shield has the temperature of that shield. The presure in the LVI layers is assumed to be so low (< 13.3 mPa) that molecular transport in the system does not play any appreciable role [4, 6]. In the assumptions made here the heat-balance equations for the cooled shields have the form

$$q_{i+1} - q_i = Gc_p (T_i - T_{i-1}), \ i = 1, \ \ldots, \ N, \tag{1}$$

where q_i is the density of the heat flux through the LVI in the i-th interval, formed the (i − 1)-th and i-th cooled shields and T_i is the temperature of the i-th shield [the cold and hot boundary surfaces are assumed to be the 0-th and (N + 1)-th shields].

The heat-flux density is

$$q_i = \Lambda_i (T_i - T_{i-1}), \tag{2}$$

where Λ_i is the thermal conductivity of the i-th interval, calculated from the formula

$$\Lambda_i = \begin{cases} \bar{\lambda}_i / l_i, & \rho_i \neq 0, \\ \varepsilon_0 \sigma (T_{i-1} + T_i)(T_{i-1}^2 + T_i^2), & \rho_i = 0. \end{cases} \tag{3}$$

In turn, the mean-integrated thermal conductivity of the LVI stack is

$$\bar{\lambda}_i = a (T_{i-1} + T_i)(T_{i-1}^2 + T_i^2) + \frac{b}{h_i} \bar{\lambda}_m (T_{i-1}, \ T_i), \tag{4}$$

where h_i is the distance between the LVI layers in the i-th interval in the stack (i = 1, ..., N + 1); $\bar{\lambda}_m$ is the mean-integrated thermal conductivity of the material of the insert, and a and b are coefficients that depend, respectively, on the characteristics of the heat shields (degree of blackness, perforation) and the inserts of the given LVI. Formula (4) is derived by integrating the theoretical-empirical correlation obtained in [3] for the transverse thermal conductivity of the LVI stack,

$$\lambda_{\perp} (T) = 4aT^3 h + \frac{b}{h} \lambda_m (T). \tag{5}$$

In the steady-state regime the flow rate of the exhaust cryogen vapor satisfies the equation

$$\delta (G) \equiv \frac{q_1 (G; \ \mathbf{l}, \ \mathbf{h})}{r} - G = 0, \tag{6}$$

where $\mathbf{l} = (l_1, \ \ldots, \ l_{N+1})$ and $\mathbf{h} = (h_1, \ \ldots, \ h_{N+1})$.

The root of Eq. (6) corresponds to self-balancing flow rate $G_* = G(\mathbf{l}, \mathbf{h})$.

The system under consideration is optimized in order to minimize the heat influx into the cryostating zone,

$$q_1 = q_1 (G_*, \ \mathbf{l}, \ \mathbf{h}). \tag{7}$$

The role of the variable parameters in this case is played by vector $\mathbf{l}$, and h. The components of the first vector obey the condition

$$\sum_{i=1}^{N+1} l_i = L, \ l_i \geqslant l_0. \tag{8}$$

We note that in this problem the number of variables can be halved by eliminating the vector h. The optimum values of the components of this vector are found analytical as the minimum points of expressions (4):

$$h_{0i} = \left[\frac{b \bar{\lambda}_m}{a (T_{i-1} + T_i)(T_{i-1}^2 + T_i^2)} \right]^{0.5}. \tag{9}$$

If it turns out that for some interval the packing density of LVI layers per cm, $\rho_i = 10^{-2}/h_{0i}$, is smaller than the minimum allowable packing ρ_0, then in this case ρ_i can be defined in one of two variants: a) $\rho_i = 0$ and heat transfer between the cooled shields occurs only through radiation; b) $\rho_i = \rho_0$, i.e., the packing density is assumed to be equal to the minimum allowable value and $h_{0i} = 10^{-2}/\rho_0$. The better of these two possibilities is realized, i.e., the one for which the conductiity of the interval under consideration is lower.

After this the algorithm for calculating the objective function at a fixed consists of the following.

At a given flow rate G the system of equations (1)-(4) is reduced to nonlinear three-point equations

$$A_i T_{i-1} - D_i T_i + B_i T_{i+1} = 0, \qquad (10)$$

where $A_i = \Lambda_i + Gc_p$, $B_i = \Lambda_{i+1}$ and $D_i = A_i + B_i$, $i = 1, \ldots, N$. System (10) in the temperatures of the shields is solved by the method of scalar implicit differencing [7] combined with iterations over nonlinear ities. The coefficients in (10) in this case are calculated from the values of the temperature from the previous iteration. The values $T^{(0)}_i = T_h$, $i \geq 1$ serve as the initial approximation. After calculating the temperature with the desired accuracy we find the heat flux $q_1(G, l)$ and the discrepancy $\delta(G)$. From physical considerations it is clear that δ is a continuous, monotonically decreasing function of the flow rate and $\delta(0) > 0$ and $\delta(\infty) < 0$. We calculate the unique solution of Eq. (6) using the iterative method of false positions [8]. The value of the objective function is defined as the heat in influx the cryostating zone in the case of a self-balancing flow rate. The minimum of the objective function $\min\limits_{l} q_1(G_*, l)$ is found by the penalty function method.

We note that the above algorithm is also applicable to the problem in which the shields are attached to a nozzle that is ideally cooled by exhaust vapor. In this case instead of (1) we consider the equations

$$q_{i+1} + \frac{Gc_p}{S} \frac{T_{i+1} - T_i}{1 - \exp\left(-\frac{Gc_p}{\lambda_n F_n} l_{i+1}\right)} = q_i + \frac{Gc_p}{S} \frac{T_i - T_{i-1}}{\exp\left(\frac{Gc_p}{\lambda_n F_n} l_i\right) - 1},$$

where λ_n is the thermal conductivity of the material of the orifice, F_n is the cross section of the orifice, and S is the area of the boundary surface.

The calculations were carried out for an LVI sample consisting of a perforated corrugated PET (polyethylene terephthalate) film metallized on both sides with inserts of ÉVTI-7 fiberglass mat. The tabulated dependence of the thermal conductivity on the temperature for the material of the inserts (Pyrex glass) is approximated to within 3% in the range 4-350 K by the polynomial

$$\lambda_m(T) = 9.456 \cdot 10^{-2} + 3.285 \cdot 10^{-3} T + 2.494 \cdot 10^{-5} T^2 - 1.505 \cdot 10^{-7} T^3 + 2.238 \cdot 10^{-10} T^4.$$

The parameters $a = 1.89 \cdot 10^{-9}$ and $b = 1.96 \cdot 10^{-8}$ in (5) were determined by the method of least squares. In this case the functions T^3 and $\lambda_m(T)$ were used as the basis functions and the experimental data of [2] for the LVI sample under consideration with a packing density of 31 shields/cm were used as the results of measurements. The reduced degree of black-

TABLE 1. Comparison of the Calculated Variants

No. of variant	1	2	3	4
Heat influx into the cryostating zone q_1, mW/m^2	4,18* 5,81	4,82 7,18	5,74 8,34	6,10 8,77
Relative deterioration %	0 0	15 24	37 44	46 51

*The values in the upper row correspond to N = 4 and those in the lower row, to N = 2.

TABLE 2. Results of Optimization of LVI/Cooled-Shields System at N = 4

i	No. of variant of calculation								
	1			2			3		
	l_i, mm	ρ_i, cm^{-1}	T_i, K	l_i, mm	ρ_i, cm^{-1}	T_i, K	l_i, mm	ρ_i, cm^{-1}	T_i, K
1	10	0	39	20	0	40	11	20	16
2	11	10	86	20	10	115	12	20	48
3	22	11	159	20	13	186	17	20	109
4	23	17	221	20	20	246	25	20	199
5	35	26	—	20	27	—	35	20	—

TABLE 3. Results of Optimization of LVI/Cooled-Shields System at N = 2

i	No. of variant of calculation								
	1			2			3		
	l_i, mm	ρ_i, cm^{-1}	T_i, K	l_i, mm	ρ_i, cm^{-1}	T_i, K	l_i, mm	ρ_i, cm^{-1}	T_i, K
1	10	0	42	33	10	56	22	20	32
2	33	10	150	33	11	179	29	20	129
3	57	23	—	34	24	—	49	20	—

ness for neighboring cooled shields was taken to be equal to the reduced degree of blackness of the neighboring shields of the LVI: $\varepsilon_0 = a/\sigma \approx 0.033$. The other parameters were L = 10 cm, ℓ_0 = 1 cm, ρ_0 = 10 cm^{-1}, T_c = 4.2 K, T_h = 300 K, r = $2.06 \cdot 10^4$ J/Kg, and c_p = 5200 $J \cdot K^{-1}/kg$.

We consider the following variants:

1) both the arrangement of the cooled shields and the packing density of the LVI layers are optimized;

2) the packing density of the LVI layers is optimized with the cooled shields arrayed equidistantly;

3) the arrangement of the shields is optimized with a given uniform arrangement of the LVI layers: ρ_i = 20 cm^{-1} ($i \geq 1$);

4) the shields are arranged equidistantly and the packing density of the LVI layers is uniform (ρ = 20 cm^{-1}).

The results of the calculations are given in Tables 1-3.

From Table 1 we see that the worst variant of a heat-insulating system is the one with equidistant cooled shields and uniformly packed LVI in all the intervals. When uniform LVI packing is maintained optimiztion of the shield arrangement considerably worsens the efficiency of the heat-insulating system. The system is improved substantially, however, by optimization of the packing and to an even greater extent by optimization of both the LVI packing and the arrangement of the cooled shields. The efficiency of the system also depends significantly on an increase in the number of cooled shields. Thus (see variant 1) the heat influx into the cryostating zone with four shields is almost 40% lower than in the case with two shields.

An analysis of Tables 2 and 3 indicates that it is inadvisable to use LVI in the interval between the cold boundary surface and its neighboring shield if the temperature of the shield is below 50 K. It is recommended that the minimum interal allowable by design considerations be chosen in this case.

NOTATION

q, heat flux density; T, temperature; G, c_p, and r, mass flow, specific isobaric heat capacity, and heat of vaporization of the cryogen; N, number of cooled shields; ℓ and ℓ_0, shield separation and the minimum allowable shield separation; ρ and ρ_0, VLI packing density and the minimum allowable packing density; L, sum of the intervals between shields (including the boundary surfaces); h, distance between LVI layers; Λ, thermal conductivity of an interval; λ_m, mean integrated thermal conductivity; ε_0, reduced degree of balckness of neighboring shields; σ, Stefan–Boltzmann constant; and the subscript is the number of an interval or a cooled shield.

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CONTRIBUTION TO THE THEORY OF THE VISCOELASTICITY OF DISPERSE SYSTEMS UNDER THE CONDITIONS OF HEAT AND MASS TRANSFER

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An open system of equations for the simultaneous description of heat and mass transfer and deformation processes in disperse materials is derived on the basis of the Boltzmann–Volterra theory and the first law of thermodynamics for open systems.

Introduction. The problem of the interplay of heat and mass transfer and deformation phenomena is of unquestionable interest from the theoretical and applied standpoints. It is important, e.g., in problems of the optimization of the technological moisture-heat treatment processes, including drying of disperse materials under loading and deformation. Modern science knows of a number of theoretical methods for taking account of the effect of heat and moisture on the deformation properties of materials. Within the framework of the hereditary Boltzmann–Volterra theory [1] the effect of heat and moisture on creep of materials and stress relaxation in them is usually taken into account by the method of factor-time analogies [2, 3] in qualitative agreement with experiment. It seems more consistent, however, not to make a one-sided allowance for only the effect of heat transfer on the rheological processes but rather to describe their effect on each other. Clearly, such a complex description requires the invocation of not only mechanical laws but also thermodynamic laws and their interaction.

In this communication we propose a variant of the complex description of the above-mentioned phenomena on the basis of the hereditary Boltzmann–Volterra relations and the first law of thermodynamics for open systems, using a number of model relations. As a result we obtain a system of equations for the concurrent description of the heat and mass transfer and deformation processes and make a preliminary analysis of a number of its general results.

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